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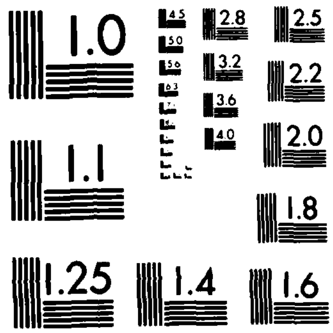

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SOME APPLICATIONS OF FUZZY SETS AND THE ANALYTICAL HIERARCHY PROCESS TO DECISION MAKING

by

Alberto Castro Rosas

September 1984

Thesis Advisor: G.F. Lindsay

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This thesis examines the use of fuzzy set theory and the analytic hierarchy process in decision making. It begins by reviewing the insight of psychologists, social scientists and computer scientists to the decision making process. The Operations Research-Systems Analysis approach is discussed followed by a presentation of the basis of fuzzy set theory and the analytic hierarchy process.

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Two applications of these methods are presented. The first uses fuzzy sets and a little of the analytic hierarchy process to solve an hypothetical decision problem for the commanding officer of a naval task force. The second applies the latter technique and estimated data to the problem of choosing the best alternative to provide quality air service to Mexico City.

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Some Applications of Fuzzy Sets and the Analytical Hierarchy Process to Decision Making

by

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Commander, Mexican Navy
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Submitted in partial fulfillment of the requirements for the degree of

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This thesis examines the use of fuzzy set theory and the analytic hierarchy process in decision making. It begins by reviewing the insight of psychologists, social scientists and computer scientists to the decision making process. The Operations Research-Systems Analysis approach is discussed followed by a presentation of the basis of fuzzy set theory and the analytic hierarchy process.

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I. INTRODUCTION

A. GENERAL OVERVIEW

It has been said that the countries of the third world, more than being underdeveloped are underadministrated and it has been said also, that the heart of management is leadership and the heart of leadership is decision-making. Since Decision Theory is considered a subset (in some instances is considered synonymous) of Operations Research, the author of this paper is convinced that a survey of the decision process will be very useful not only for himself but also for the institution to which he belongs.

The decision process (perhaps as a measure of its importance) has been examined from several points of view, which we will try to review briefly. In order to do this, in this chapter we will discuss the psychologists' and social scientists' insight into the decision process. Then we will review some of the topics of Computer Science related to the decision process.

In Chapter Two, we will present the differences between, and common characteristics of, Operations Research and Systems Analysis in solving decision-making problems. We will mention <u>some</u> of the available techniques, and we will present a tentative classification of them. There, we will discuss at some extent the issue of how a decision maker can handle decision-making problems under certainty, risk or uncertainty.

In Chapter Three, we will present some ideas, notation and definitions of fuzzy set theory and in Chapter Four we will present the main ideas of the analytic hierarchy process, which we will use later on.

Two cases of decision-making problems will be presented, one in in Chapter Five and the other in Chapter Six. The first is an application of fuzzy set theory to an hypothetical naval decision; the second case will use the analytical hierarchy process to give another insight to a problem related with the airport of Mexico City. Finally, in Chapter Seven we will comment on the differences and similarities between the two approaches used in the previous chapters.

B. THE PSYCHOLOGIST'S POINT OF VIEW

Psychologists are concerned with the way the human being performs the subjective function of decision-making; how he combines knowledge, memory, experience, information, feelings, etc., and, using his reason (and sometimes his instinct), reaches a decision when faced with a problem which requires a solution, or a choice, or in general when he receives a stimulus which requires a response in the way of some action. A model of that process is given by Kokawa in [Ref. 1] and is presented in Figure 1.1.

Another point of view is that of Kellerman [Ref. 2] whose analysis is related with the conflicts, needs and personality traits of the decision maker, that is:

i). It has been found that the decision maker usually is subject to an intrapersonal conflict when he has to chose between two alternatives. When the alternatives are equally attractive, the conflict is called an approach-approach conflict. If the alternatives are equally repulsive, the decision maker will be in an avoidance-avoidance conflict. When the decision maker is in a "go - no go" situation, because the proposed course of action has attractive and painful aspects, his conflict is of the approach-avoidance type. The more common type of conflict that arises is when the decision

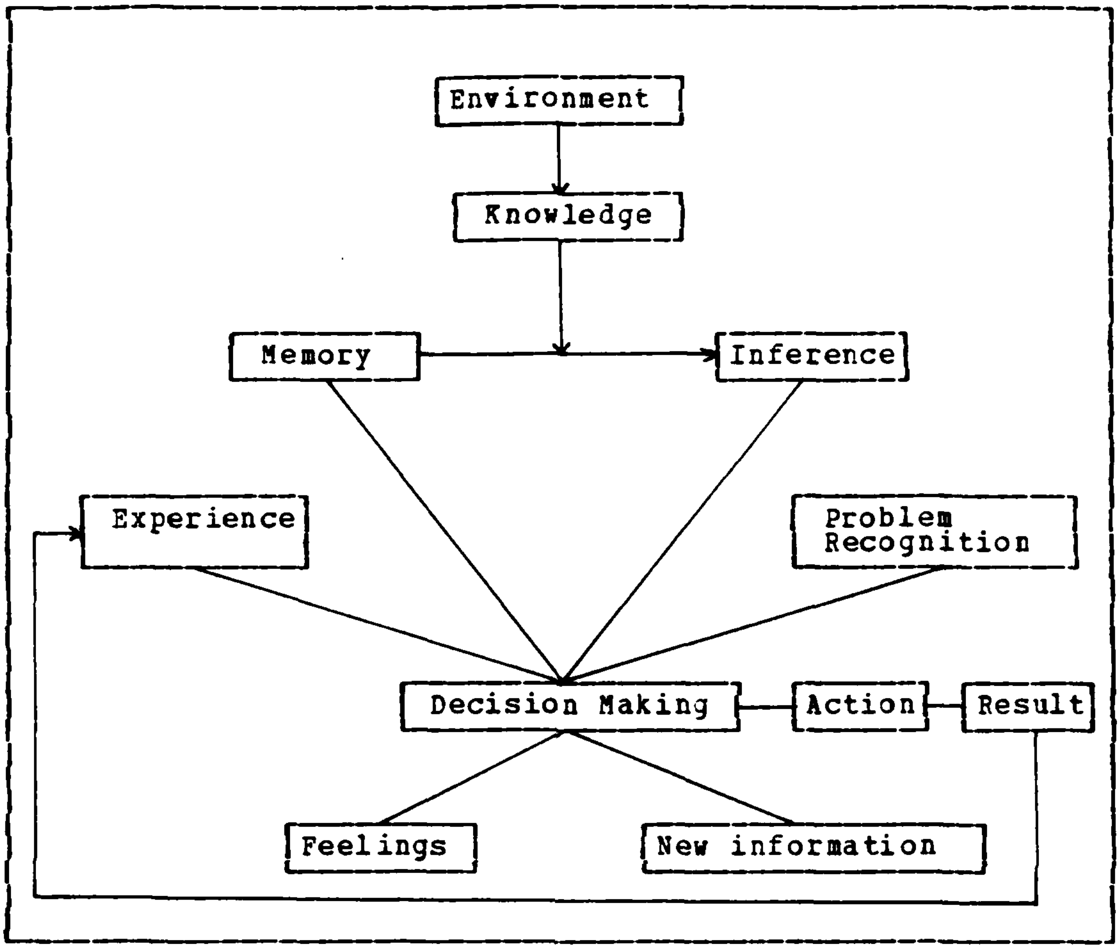

Figure 1.1 The Human Decision-Making Process.

maker has two or more alternatives which have both attractive and repulsive aspects.

ii). Two important decision maker's needs are simplicity and consistency. The first is the tendency to develop models which are simpler and more manageable than the complex and overwhelming problem with all its details, i.e., the human being filters out the countless stimuli he receives excluding those he believes are unimportant (sometimes erroneously). The other need is the consistency between attitudes and behavior, i.e., if the human

being behaves in a way contrary to his attitudes, a stress condition is developed.

iii). The personality traits that influence the decision making process and differ in each individual are the following: His tolerance for ambiguity or his willingness to deal with problems whereby the outcomes are random, or vague, or where there simply there isn't enough information to use. Here a person with high tolerance for ambiguity is more likely to be patient in evaluating or collecting information before taking action.

On the contrary, a decision maker who is intolerant of ambiguity, will tend to take action prematurely to avoid the stress the ambiguity causes him. Raiffa, in [Ref. 3] pp. 159, says "....the fact is, most people are willing to pay excessive amounts of money to get rid of vagueness....".

Another trait is the decision maker's self concept, which can be negative or positive. This trait should affect the decision making process in two ways: the decision maker with a negative self-concept experiences more anxiety making a decision than another with a positive self-concept, and the first will behave trying to conform his actions to another's beliefs rather than his own.

Another trait to be considered is the so called locus of control, in which individuals differ in their perceptions of the control they have over the possible outcomes. At one extreme, there are individuals who believe that all outcomes depend upon their behavior, while (on the other extreme) others believe that the outcomes are determined by fate or luck. Most people are in the middle of these extremes.

Finally, the last personality trait to be mentioned (perhaps the more important from our personal point of view) is the decision maker's willingness to take risks, and since we will dwell on that issue later on, we will stop at this point from the psychologist's point of view.

C. THE SOCIAL SCIENTISTS' POINT OF VIEW

Social scientists have directed their efforts to the decision-making process in organizations and other social systems. They are concerned with descriptive and prescriptive theories about how organizations make their choices, the interaction between the organizational structure, and the forms of choice.

In this regard it is worth mentioning the Garbage Can Model of organizational choice initiated by Cohen, March and Olsen [Ref. 4] which in general terms states that some organizations (or all at some degree or under some circumstances) can be characterized by:

i) It is difficult to set up and define a set of preferences to the decision situation that satisfies the standard consistency requirements for a theory of choice (we will treat in more detail the consistency issue later on).

ii) There is a lack of understanding by the members of the organization of the processes, goals, and objectives of the organization.

iii) The members have a great deal of mobility, i.e., they remain a short time with the organization.

Such organizations are called organized anarchies and the decision-making process is thought of as choices looking for problems, problems looking for solutions and decision makers looking for work, all of them in a garbage can. A simulation model was set up to examine the forms of choice

(by resolution, by flight or by oversight) as a function of the structure of the organization. In the proposed model all inputs are deterministic and the structure and some coefficients are varied. Recent works have introduced the Monte Carlo method to represent uncertainties and to improve the model. Since the conditions that are supposed to exist in organized anarchies fit very well to many organizations, this theory has gained a great deal of attention and there is active research in this field to improve the model and to use it in the design of organizations.

D. THE COMPUTER SCIENTIST'S POINT OF VIEW

Another discipline which has focused on the decision process is Computer Science and its insight is twofold: One is Artificial Intelligence and the other is the design of Decision Support Systems.

1. Artificial Intelligence

Artificial Intelligence is subdivided in two branches of study: The engineering approach and the modeling approach. The first includes topics as pattern recognition, translating text from one language to another, composing music, motion learning (which is already widely used in industrial robots) and others, while the modeling approach includes simulations of human problem solving and decision making processes, and learning behavior by building models of neural networks [Ref. 5].

2. Decision Support Systems

Decision Support Systems include a great variety of applications and as Alter [Ref. 6] says "...statements about Decision Support Systems as some kind of homogeneous category of things should be subject to great scrutiny....". We can mention the following generic types:

Management Information Systems which include applications at a clerical level, i.e., processing transactions as sales orders, billing and receipts, payables, and inventory accounting. These functions support basic operations in accounting, marketing, manufacturing, etc. Alter compares them with a file drawer, and in other contexts they are called Information Systems and are used mainly to support operational control. Another type of Management Information System is Information and Data Analysis which allows one to extract relevant data from databases, maintaining and/or presenting this data in a form suitable for standard or ad-hoc analysis, which will aid managers to measure performances, reallocate resources, formulate new policies, etc. The last application to be mentioned (because we cannot be exhaustive) in Management Information Systems is the accounting system which maintains the information used to measure and report the resources that flow into or out of the enterprise and allows the formulation and/or estimation of the financial reports (Income Statement, Balance Sheet and Statement of Changes in Financial Position). The purpose of formulating these reports is to communicate to external parties the results of operations, the financial position, and the flow of funds in the enterprise. The purpose of estimation is to forecast the consequences of policies, controls, or performances and aid in their design, analysis, and planning.

Another generic type of Decision Support Systems can embrace systems which perform functions such as simulation, optimization, or aiding strategic planning.

Simulation is defined as ".... a controlled statistical sampling technique (experimental) which is used, in conjunction with a model, to obtain approximate answers for complex (probabilistic) problems when analytical and numerical techniques cannot supply answers...." [Ref. 7].

Although we can have "physical simulation", we are refering to simulation performed on a digital computer. Examples of applications of simulation are problems related to job-shops, queues, prediction of the performance of computers, system-design concept evaluation, system-destruction or safety experiments (where the experiment is too dangerous to be performed physically), system reliability or failure testing (economically unfeasible) or in general testing systems too complex and too large which are difficult if not impossible to do physically, e.g., spacecraft maneuvering, large man-machine systems, weapons effects, etc. Simulation requires the use of pseudo-random number generators or general purpose simulation languages such as SIMSCRIPT, GPSS and SIMULA which simplify the simulation work.

Optimization systems are mainly "off the shelf" or customer tailored computer software packages or codes which solve problems with well-behaved objective function and constraints. The more widely known and used are those which optimize using linear, non linear or other mathematical programming techniques.

The last category of Decision Support Systems we want to mention is the computer based Combat Systems such as the SAGE (Semiautomatic Ground Environment System, developed by the US Army in 1958), the NTDS (Navy's Tactical Data System) and the AEGIS System. Following we give a brief description of the latter. This system is an update of the second and is composed of the weapon control system Mark 1, the fire control system Mark 99, the guided missile launching system Mark 26, the operational readiness test system Mark 1, the phased array radar system (AN/SPY-1A), the display group and the Command and Decision System Mark 1. The entire system coordinates functions as air and surface radar search, identification, electronic warfare, navigation, underwater surveillance, target acquisition and

tracking, and controls the gun weapons system and the Harpoon and Tomahawk systems in a sophisticated type of Local Area Network with 35 processors capable of backing up some of them. The system has 15 AN/UYK-7 processors and 20 AN/UYK-20.

It is interesting to note that in our discussion, the trend has been to move from a type of analysis intended by psychologists and social scientists which is mainly of the descriptive type to a more normative decision analysis such as that intended by the computer scientists. The former try to explain people's beliefs and preferences as they are, not as they should be. On the other hand, the latter is concerned with the rules that a decision maker must follow to optimize the expected consequences of actions taken in a choice situation and to insure the coherence of beliefs and preferences. This trend will be stressed in the next chapter where we will review only normative decision analysis as those used in Operations Research and Systems Analysis.

II. THE OPERATIONS RESEARCH-SYSTEMS ANALYSIS APPROACH

A. GENERAL OVERVIEW

The opinions about the scope of these disciplines are divided. In fact, Quade in [Ref. 8], says, that both must be included in a more comprehensive field which he calls Policy Analysis. Also, he states that the difference between them is a matter of level of applications more than of method, i.e., Operations Research deals with efficiency problems while Systems Analysis is used in optimal choice problems and consists mainly of cost-effectiveness and cost-benefit studies. For Quade, Policy Analysis includes four types of projects:

1) Improvement in efficiency of operations where the analyst models the situation using techniques like simulation, linear programming, or queueing theory and looks for the optimal output of the operation being studied. This area seems the more appropriate for the techniques of Operations Research.
2) Problems of resource allocation in which the decision-maker (or the analyst who works for him) must find out the optimal allocation of funds among competing programs.
3) Program evaluation consisting of the measurement of the effectiveness of ongoing programs and the identification of strategies and policies which are considered (or found out to be) causes of the behavior of that program.
4) Planning and budgeting activities, which are a very specific type of resource allocation performed by government agencies at several levels in order to determine the objectives or goals specifying the best way to achieve them.

Another point of view of these disciplines, but now restricted to Defense applications, is that of Dr. Alain Enthoven [Ref. 9] who says:

> "....What is Operations Research or Systems Analysis at the Defense policy level all about?. I think that it can best described as a continuing dialogue between the policy maker and the systems analyst, in which the policy maker asks for alternative solutions to his problem, make decisions to exclude some, and make value judgements and policy decisions, while the analyst attempts to clarify the conceptual framework in which decisions must be made, to define alternative possible objectives and criteria, and to explore in as clear terms as possible (and quantitatively) the cost and effectiveness of alternative courses of action. The analyst at this level is not computing optimum solutions or making decisions. In fact, computation is not his most important contribution. And he is helping someone else to make decisions. His job is to ask and find answers to the questions: "What are we trying to do?","What are the alternative ways to achieve it?","What does the decision maker need to know in order to make a choice?", and to collect and organize this information for those who are responsible for deciding what the defense program cught to be....".

However a word of warning must be given at this point about over-emphasising the usefulness of the O.R.-Systems Analysis approach to avoid thinking of them as a panacea that will solve all types of problems (although we will cover a new insight called Analytical Hierarchy Process which can be used in almost all types of problems). In fact, we can menticn two examples which illustrate this point. One is given by Quade [Ref. 8] who, on page 103, quotes the statement of the House Armed Services Committee (1536, 16 May 1966) which remarked of the Defense Department that their "....Almost obsessional dedication to cost-effectiveness, raises the specter of a decision maker who....knows the price of everything and the value of nothing....". Other critics of the approach with which we are dealing is Summers [Ref. 10] who, analyzing the extensive use of System Analysis in the Department of Defense following its introduction by McNamara, and regarding the Vietnam war, says "....Systems Analysts ignore the

irreconcilable conflict between any system or model which has the finite nature of a synthesis, and the conduct of war, which branches out in almost all directions and has no definite limits, and so, they have an educated incapacity to see war in its true light....". In page 45 he states also, about Systems Analysis, that "....While it was efficient in structuring forces in preparation for war, it was neither designed for, nor was it capable of fighting the war itself...", but the officials in the higher positions in the Department of Defense were from that school.

But in spite of this criticism and because analytical techniques are, by far, a better choice instead of using habit, snap judgement, impulse or just plain chance (the toss of a coin) in making critical and even routine decisions, we will concentrate now on some of these techniques which can assist the decision maker (not to replace him) and will help him avoid relying on "gut feel" alone.

The concentration we spoke about before is a constraint we need, because from a broad point of view, every action results from a decision, so that almost every theory which involves taking action would be a decision theory; so we will try to glance through some of these analytical techniques, classifying them according to several criteria.

B. CLASSIFICATION OF ANALYTICAL TECHNIQUES

We have said at the end of Chapter I that the most straightforward classification of decision analysis techniques, is to divide them in descriptive and normative. In this section we are going to classify those techniques included in the latter type, following Kickert [Ref. 11].

1. <u>Number of Decision Makers</u>

According to the number of decision makers we have the single person decision-making problem, which is studied by the Statistical Decision Theory and the Analytical Hierarchy Process, while two-person decision making and multi-person decision making are addressed by the two person Game Theory (zero and non-zero sum games) and the n-persons Game Theory, respectively. This last is addressed by theories of group and team decision-making.

2. <u>Amount of vagueness</u>

Another dimension reflects the quantity, quality and reliability of the information we have of the situation with which we are dealing. Generally, this insight is covered by what is called Risk Analysis.

It is convenient at this point to give some notation and definitions that will allow us to be more precise in our review of the decision problem for a single decision maker.

There is a set A={A(1),A(2),....A(n)} of alternatives or courses of action, which are mutually exclusive and (hopefully) exhaustive, available to the decision maker.

The possible states of nature is a set S={S(1),S(2),..S(m)}, that is to say, events that are out of control of the decision maker, but that are not considered to be in contention with him, or in other words, the decision maker has no rational opponent that reacts against him.

To each alternative A(i) and each state S(j) correspond an outcome R(i,j) that represents the gains (or losses) that the decision maker will obtain if he follows alternative A(i) and the state S(j) occurs.

If an alternative A(k) has all outcomes R(k,j) greater or equal than the corresponding outcomes R(i,j) of

alternative A(i), then we say that A(k) dominates A(i), and in that case this last alternative can be neglected.

In this setting, a _Decision is defined as a choice between alternatives_, and the process can be represented as a matrix whose entries are the outcomes R(i,j), with the set A heading the rows and the set S heading the columns. According to the amount of vagueness we have three situations: certainty, risk and uncertainty.

Certainty refers to the situation where all information is deterministic, or in other words there is only one possible state with no random variables or stochastic processes involved .The decision making problem consists of an optimization of a function of n variables, subject to one or several constraints.

Risk describes the case where, with the information we have at hand, we can assign probability P(j) to the occurrence of state S(j). In this case we can use some frequently used principles of choice [Ref. 12].

The Expectation principle states that we select the alternative that maximizes the expected profit or minimizes the expected cost, i.e.,

$$A(optimal) = \max_i \sum_j \{R(i,j)*P(j)\} \quad i=1,2,...n.$$

The Most Probable Future principle states that we must treat the state that has the highest probability of occurrence as the sure event and solve the problem as one under certainty.

The Expectation-Variance principle suggest that sometimes a medium-valued expected return with small variance is prefered to the maximum expected return with greater variance.

The Aspiration Level principle simply states that if the decision maker has some threshold (minimum if he is

looking for gains or maximum if he is dealing with costs), then he can maximize the probability of achieving his aspiration described by that threshold.

It is here precisely where the main idea of this paper comes into place because this assignation of probabilities can not, in some cases be made in a mathematical and objective (or even subjective) way. This can be due to a lack of information (situation frequently encountered in the underdeveloped countries), or to an ill-definedness, inexactness, or in short, fuzziness of the situation at hand. Sometimes the alternatives are defined by the decision maker in terms of desires, using words as "more or less....", "reach a very high level of....", etc.. It means that there are situations where we have doubts of the exactness of concepts, correctness of statements and judgements and degrees of credibility in which case, the assignment of the probability of occurrence of one event has no (or almost no) meaning.

It is in this case where the *Fuzzy Set Theory*, initiated by Zadeh [Ref. 13], comes to aid in handling this type of situation, since it is defined as a mathematical theory of vagueness.

Uncertainty When the probabilities of occurrence of future states are unknown (or the decision maker is unwilling to assign them), we can use again certain principles of choice [Ref. 12].

The principle of insufficient reason (or Laplace principle) consists of assigning the same probabilities to all possible alternatives and choosing the alternative whose average return is most favorable.

The Pessimistic (or Maximin gain) principle states that the decision maker should choose to make the worst outcome as good as possible, by choosing first the minimum payoff for each alternative (i.e. the minimum value in each

row) and then choose the alternative that has the highest value among the payoffs selected previously, or:

$$A(\text{optimal}) = \underset{i}{\text{Max}} \{ \underset{j}{\text{Min}}\ R(i,j) \}.$$

In case the payoffs are costs, then the pessimistic principle is Minimax, that is:

$$A(\text{optimal}) = \underset{i}{\text{Min}} \{ \underset{j}{\text{Max}}\ R(i,j) \}.$$

The Optimistic principle (Maximax for gains) states that the decision maker must choose that alternative which has the highest payoff, that is

$$A(\text{optimal}) = \underset{i}{\text{Max}} \{ \underset{j}{\text{Max}}\ R(i,j) \}.$$

The Hurwicz principle uses an index of optimism, a, (between 0 and 1) and, for gains, has the formula:

$$A(\text{opt.}) = \underset{i}{\text{Max}} \{a * \underset{j}{\text{Max}}\ R(i,j) + (1-a) * \underset{j}{\text{Min}}\ R(i,j)\}.$$

and for costs the formula

$$A(\text{opt.}) = \underset{i}{\text{Min}} \{a * \underset{j}{\text{Min}}\ R(i,j) + (1-a) * \underset{j}{\text{Max}}\ R(i,j)\}$$

The last principle of choice to be reviewed under uncertainty is Savage's Principle of Minimax Regret in which the decision maker follows the procedure: If he is dealing with gains, transform the payoff matrix, assigning by columns, 0 to the highest value of that column and substituting each of the other entries by the difference between the highest value of that column and the entry's value. This produces a matrix of "regrets", and using it he chooses using the Minimax principle.

3. Mathematical Tools

Another criterion we will use to classify the analytical techniques, is related to the mathematical tools we can use to solve decision making problems, and this includes maximization of expected utility, constrained optimization and multiple optimization.

Maximization of expected utility assumes that there exists an idealized decision maker, who extracts all the information contained in available evidence, has a coherent set of values, and is able to find the optimal course of action. It is addressed by Statistical Decision Theory and was initiated mainly by Savage, who in [Ref. 14], was the first to question the classical statistical insight and began the now-called Bayesian insight. The other fundamental source was that of Von-Neumann and Morgenstern [Ref. 15] where the axioms of the utility function were developed. This insight is widely known [Ref. 16], [Ref. 3], [Ref. 17]. The most common technique uses a decision tree, which with Bayes' formula and expected value or expected utility, allows us to select the best alternative.

The utility function of a risk-averse person is concave and lies above the x = y line, which, by the way, stands for those who make their choices according to the expected value of the outcomes. On the other hand, the utility function of a risk-seeking person is convex, and lies below the x = y line.

This function is determined eliciting information from the decision-maker in the form of choices between two gambles, or between a gamble and a sure gain or loss.

This determination, however, has been found to be subject to some biases, mentioned by Tversky and Kahneman in [Ref. 24]. One of the biases is the overweighting of the probabilities when they are in a range near zero, while they are underweighted when they are near 1.

The second bias is called "framing" by the authors, and will be explained quoting the example they use:

> "....Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs are proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:
> If program A is adopted, 200 people will be saved.
> If program B is adopted, there is a 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.
> The majority choice in this problem is risk-averse: the prospect of certainly saving 200 lives is more attractive than a risk prospect of equal expected value....Another version of the problem is...:
> If program A' is adopted 400 people will die
> If program B' is adopted there is a 1/3 probability that nobody will die and 2/3 probability that 600 people will die.
> In this version, a large majority of respondents exhibited a risk-seeking preference for adoption B' over A'...."

Both versions basically refer to the same situation and this contradiction is caused by the framing effect. What we want to emphasize is the need to be aware of these biases when using Bayesian decision theory.

Optimization under constraints consist of optimizing well defined objective functions subject to constraints and is perhaps the best known and most used of all decision theories and includes Linear Programming (initiated by Dantzig), Non-linear Programming (where the work of Kunh-Tucker was the major improvement since Lagrange's time) and other techniques such as Dynamic, Integer and Combinatorial Programming and Network Flows

Multiple optimization is addressed by techniques such as multi-person and multi-criteria optimization, where we have for the former n-person Game Theory, Group Decision Making (which branches out according to the size of the group) and Team Decision Making (mainly studied in the Command, Control and Communications environment), while the latter is treated by the Maximum Vector Theory.

4. Decision Making Using Fuzzy Sets.

We have already mentioned that one of the main ideas driving this paper is the use of Fuzzy Sets in the Decision-Making process. Even though it is a relatively new field (the pioneer work was written by Zadeh in 1965) there exists an active research effort in the area and since it is not well known, we will present the main ideas in the next chapter.

5. The Analytical Hierarchy Process

One of the workshops sponsored by ORSA and presented previous to the Joint National Meeting TIMS/ORSA of 1984 concerned the Analytical Hierarchy Process, which according to [Ref. 18] "....provides a workable approach to the most complex issues and problems,--problems characterized by multiple criteria, ambiguity and conflicting interests, and problems which must address both qualitative and quantitative information....". We will cover in a later chapter the main ideas of this new technique for Decision Making.

Our intention has been to present a broad perspective of the techniques used in Operations Research for solving some decision problems. In the next chapter we will examine the notion of fuzzy sets as a decision aid.

III. BRIEF REVIEW OF THE THEORY OF FUZZY SETS

A. EXPLANATION

New ideas can be thought of as milestones which mark the progress of mankind, because whenever a new idea comes up, mankind takes a step forward. New ideas do not receive the welcome they should. First, they must contend with or integrate within "common sense" or generally accepted "truths". Second, there exists a human trend to resist change. It is usual, for example, to contrast new ideas with some perfect, unattainable standard, instead of comparing them with prevailing ideas. Third, new ideas are not born polished and ready to be applied. They need some time to mature. It is in that period, however, when they are perhaps embryonic, when they receive the worst treatment. Many of these comments may be applied to the theory of fuzzy sets, which we will address in this chapter.

We will review the main ideas of fuzzy set theory, such as definitions, notation, operations and properties of fuzzy sets,but we will constrain ourselves to cover only that material that will be used later on.

B. FUZZY SETS

Since fuzzy set theory is not well known, a brief introduction to the ideas of that theory will be useful, because we are going to use them in our application of fuzzy sets to decision problems. We will present, also, some notation and definitions.

We will begin recalling [Ref. 19] the definition of a set (in the ordinary sense): A set is any collection of object which can be treated as an entity, and an object in

the collection is said to be an element or member of that set. Thus if X is a set, we can say with certainty if the element x belongs or does not belong to that set. We can use a function called membership function that has the values

$$f(x) = 1 \quad \text{if } x \text{ belongs to } X, \text{ and}$$

$$f(x) = 0 \quad \text{if } x \text{ does not belong to } X.$$

For example, if we think of X as the set of capital cities of the countries of the world, then

$$f(\text{Washington}) = 1,$$

$$f(\text{Chicago}) = 0.$$

That is, Washington does belong to that set, while Chicago does not. On the other hand,if we speak of the set of the most populated cities in the world, does Chicago belongs or not to that set?. In fact, we can find uncountable instances where the membership function <u>can not</u> take the values 1 or 0, suggesting that element belongs <u>in some degree to the set of reference</u>.

The concept of an ordinary set is fundamental in mathematics. Almost all other concepts are derived from it. Fuzzy set theory, proposed by Zadeh, enables us to handle those sets where the membership function can take any value in the closed interval [0,1]. Thus the ordinary set can be considered a particular case of fuzzy sets.

Some examples of fuzzy sets are:

The fuzzy set of the most populated cities of the world,

The fuzzy set of the numbers approximately equal to a given real number n, and

The fuzzy set of integers very near to 0.

The values of the membership function of the elements of these fuzzy sets have the following common characteristics: They are context dependent, they are subjective and are in the interval [0,1].

It is useful, at this point, to say what fuzzy set theory is not [Ref. 20].

It is not probability theory in disguise.

It is not an approximation to the truth.

It is not the result of random processes.

It is not not the result of a failure to comprehend.

We have, however, to admit that some sets do not have definite boundaries and that human beings do think and act in these terms. One of the resons because there has not been built to date a computer that can talk in plain, non programmed language with human beings, is because the computer, being a sequential machine, accepts only the logic of true or false, while the human being thinks and talks in terms of "perhaps", "I believe so", "about n", "around n",etc.

Here are the definition and main properties of fuzzy set as given by Zadeh in his pioneering work.

Let X be a space of objects with generic element denoted x, i.e., $X = \{x\}$.

A fuzzy set A in X is characterized by a membership function f(x), which associates with each point in X a real number in the interval [0,1]. In other words, we can say that A is the set of ordered pairs $\{x,f(x)\}$, such that x belongs to X, and f(x) belongs to the closed interval [0,1], where f(x) is the grade or degree of membership of x in A.

If x is not member of A, then $f(x)=0$. If x is a member of A just a little, then the degree of membership might be 0.2, that is, $f(x)=0.2$. If x is more or less a member of A, then it might be that $f(x)=0.5$. If x is strongly a member of A, then possibly $f(x)=0.8$ or finally if x is a member of A, then $f(x)=1$.

The more useful operations and properties of fuzzy sets are the following.

Inclusion: A fuzzy set A is included in the fuzzy set B if and only if

$$f_A(x) \leq f_B(x) \quad \text{for all } x.$$

Equality: The fuzzy sets A and B are equal if and only if

$$f_A(x) = f_B(x) \quad \text{for all } x.$$

Complementation: The fuzzy sets A and B are complementary if

$$f_A(x) = 1-f_B(x) \qquad \text{for all } x.$$

For example, let X be the reference set

$X = \{ x1, x2, x3, x4, x5, x6 \}$

and

$A = \{ (x1,0.13), (x2,0.61), (x3,0), (x4,0), (x5,1), (x6,0.03) \}.$

Then, the complement of A, which we will call B, is given by

$B = \{(x1,0.87), (x2,0.39), (x3,1), (x4,1), (x5,0), (x6,0.97)\}.$

Intersection: The intersection of the fuzzy sets A and B is the largest fuzzy set contained at the same time in A and B, and is equivalent to the "AND" operator, that is:

$$f_{A \cap B}(x) = \text{Min} \{ f_A(x), f_B(x) \}.$$

For example, let X = { Joe, Dan, Bob } be the set of candidates to fill a job, and A be the fuzzy set of trained candidates, i.e.,

A = {(Joe,0.4), (Dan,0.6), (Bob,0.7)}.

If B is the fuzzy set of young candidates, i.e.,

$B = \{(Joe,0.8), (Dan,0.9), (Bob,0.7)\}$,

then the fuzzy set C of candidates which are trained and young is

$C = \{ (Joe,0.4), (Dan,0.6), (Bob,0.7) \}$.

Union: The unicn of the fuzzy sets A and B is the smallest fuzzy set that contains both A and B and is equivalent to the "OR" operator, that is:

$$f_{A \cup B}(x) = Max \{ f_A(x), f_B(x) \}.$$

A useful compariscn between ordinary and fuzzy sets is that in ordinary sets, the union of two sets is represented as a circuit of two switches in parallel, while the intersection is represented as a circuit of two switches in series. Making combinations of these elementary circuits, it has been possible to build the full adder, which is the heart of the CPU of the modern computers. But if we substitute the switches we used for ordinary sets by meshes, we will have that union and intersection of fuzzy sets can be represented by the same circuits. We can then, make combinations of these operations, just as is done in ordinary sets.

Disjunctive Sum: The disjunctive sum of two fuzzy set is defined as

$$f(A \oplus B) = Max\{ Min(f(A), 1-f(B)), Min ((1-f(A), f(B))\}.$$

Difference: The difference is defined by the relation

$$f(A-B) = Min[f(A), 1-f(B)].$$

Other properties of fuzzy sets are:

$$A \cap B = B \cap A$$
$$A \cup B = B \cup A$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$
$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$A \cap A = A$$

$$A \cup A = A$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap \emptyset = \emptyset$$
$$A \cup \emptyset = A$$
$A \cap E = A$, where E is the reference or universal set
$$\bar{\bar{A}} = A$$
$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$
$$\overline{A \cup B} = \bar{A} \cap \bar{B}.$$

Thus, all the properties of ordinary sets are found in fuzzy sets, except

$A \cap \bar{A} = \emptyset$, and

$A \cup \bar{A} = E$.

An operation that we will use later on is defined as follows: Let A be a fuzzy set over the reference set X and let a>0 be a scalar. The operation of raising A to the power a, denoted A**a, gives the fuzzy set with membership function

$$f(A^a(x)) = \{f[A(x)]\}^a, \quad \text{for all } x \text{ in } X.$$

For example if we use the set

X = {Joe,Dan,Bob}

and B is the fuzzy set of young candidates

B = {(Joe,0.8),(Dan,0.9),(Bob,0.7)},

then if a=2,

B^2 = {(Joe,0.64),(Dan,0.81),(Bob,0.49)},

and if a=1/2,

$B^{1/2}$ = {(Joe,0.89),(Dan,0.95),(Bob,0.84)}.

Zadeh in [Ref. 21], associates the operation of raising to the square with the linguistic modifier of "very", that

is, B would correspond to the fuzzy set "very young candidates". It is noted, however, that if $a>1$, the effect of raising B to the power of a is to reduce the grade of membership of all the x's, but in such a way that those that have large membership values are reduced much less than those that have small values. In other words, raising B to a power greater than 1, can be regarded as making the requirement (that of being young) more stringent. On the other hand, when B is raised to a power less than 1, the membership function is increased; the smaller a is, the more the membership value is increased.

Let X and Y be sets, then the Cartesian product XxY is the collection of ordered pairs (x,y), with $x \in X$, $y \in Y$. A fuzzy relation R from x to Y is a fuzzy subset of the cartesian product, and is expressed by a two parameter membership function $f(x,y) \in [0,1]$. The concept can be generalized to a n-ary fuzzy relation which is a fuzzy subset of $X1 x X2 x \dots Xn$.

An example of a fuzzy relation is that of resemblance. Let X = {Joe, Dan} and Y={Bob, Tom}, then we can express the resemblance as:

f(Joe, Bob)=.8,
f(Joe, Tom)=.6,
f(Dan, Bob)=.2,
f(Dan, Tom)=.9

or expressed as the matrix shown below

	Bob	Tom
Joe	.8	.6
Dan	.2	.9

The concept of composition of fuzzy relations is defined as follows Let R be a fuzzy relation from X to Y and P a

fuzzy relation from Y to Z. Then the composed fuzzy relation C from X to Z is defined by the membership function

$$f(x,z) = \text{Max Min}\ \{f(x,y),\ f(y,z)\} \quad x \in X,\ y \in Y,\ z \in Z.$$

Suppose for example, that we use the relation resemblance defined in the last example, and we let Z = {John, Mike} and the fuzzy relation resemblance from Y to Z defined

	John	Mike
Bob	.3	.8
Tom	.5	.7

Then we can compose the resemblance between X={Joe, Dan} and Z={John, Mike} as follows:

	Bob	Tom
Joe	.8	.6
Dan	.2	.9

	John	Mike
Bob	.3	.8
Tom	.5	.7

	John	Mike
Joe	.5	.8
Dan	.5	.7

Other properties of, and operations with fuzzy sets enable the theory to be used in such fields as Biological Systems Theory, Analysis of Sociological Data and Phenomena, Process Control and Artificial Intelligence, but since we will not use them we are not going to review them. In the next chapter we will review the analytic hierarchy process since we will use it combined with fuzzy sets in a a decision making problem.

IV. THE ANALYTIC HIERARCHY PROCESS

The Analytical Hierarchy Process is a technique developed by Saaty, which can be used to solve complex problems of decision or planning. This chapter presents the main ideas of this method, without dwelling too much on the mathematical proofs. The interested reader can consult [Ref. 22] or [Ref. 23].

The core of the Analytical Hierarchy Process, is a procedure for obtaining a ratio scale for a group of elements, based upon a paired comparison of the elements. The procedure is as follows:

Assume we have m alternatives and we want to construct a scale, rating these alternatives according to certain criteria. What we need to do is set up a m by m matrix which will be called A, then, to compare alternative i with alternative j, assigning a value chosen from Table I, to the entry located in the ith row and jth column of A, following the rules given below.

i) If alternative i is more important than alternative j, we assign a number to a(i,j) from Table I

ii) a(j,i) = 1/a(i/j).

iii) a(i,i) = 1.0.

Saaty has shown, using the Ferron-Frobenious theorem and other results from the theory of positive matrices, that the eigenvector corresponding to the maximum eigenvalue of A, is a cardinal ratio scale for the alternatives compared.

The procedure recommended by the author in order to apply the method to a decision making problem is given in pp. 94 [Ref. 23], and since we will follow it in one of our applications, we will outline as follows:

TABLE I

The Pairwise Comparison Scale

Intensity of Importance	Definition
1	Equal importance of both elements
3	Weak importance of one element over another
5	Essential or strong importance of one element cver another
7	Demonstrated importance of one element over another
9	Absolute importance of one element over another
2,4,6,8	Intermediate values between two adjacent judgments

1. Define the problem, gathering background information in the general area under consideration. There are many ways to become informed in a subject. One can make a literature search, use the advice of paid consultants, etc.
2. Structure all the factors included in the problem in an hierarchy with as many levels as necessary. Each level must include those factors that can be compared with others, taking as criterion one factor of the immediate level above theirs. Usually, the overall objective will be in the highest level, while the alternatives will be in the bottom level. The intermediate levels will be clusters of factors related as mentioned before.
3. For each level develop a set of matrices, each matrix being the result of the pairwise comparison of the factors of that level taking as criterion one factor of the above level. We will have, therefore for each level,

as many matrices as factors there are in the next higher level. To construct each matrix we use Table I and the procedure given before.

4. The eigenvector corresponding to the maximum eigenvalue of each matrix will give us a ratio scale of the factors of that level with respect to the criterion. Those ratio scaled values are identified as priorities. Each vector of priorities is weighted by the priority of the criterion. The sum of these weighted vectors will give us the priorities of the factors of that level. In this way we can compose hierarchies with several levels.

One important feature we want to point out is that the ratio scale obtained gives us <u>a measure of the importance</u> that the decision maker assigns to each factor of a given level. This method allows us to scale factors when they don't share a common unit. It is in such situations that the subjective assignment of values in the pairwise comparison, as a measure of the importance of one factor over the other, is reasonable.

On the other hand, if the criterion is a measurable characteristic of the factors or alternatives we do not need to use any scaling method. But since the analytic hierarchy process in the general case works with several levels and the weights or priorities of each level are composed with the weights of the levels above and below it, we need to make a transformation of the obtained measures in order to be consistent with others levels of the structure where perhaps the factors are not measurable. One example of this situation would be the ranking of several armored vehicles or tanks, taking as criterion their speed on roads. It is obvious that the ranking is not a problem, since the speed can be measured. However we need to transform the speed values to another ratio scale, in order to be consistent with the method used in the analytic hierarchy process. To

do so, we can use a matrix with the entries of the diagonal equal to one and all other entries are the ratio of the speed of the tank of that row to the tank in that column. This will give us a scale ratio of the speeds which can be combined with the priorities of others levels. If we suppose that we have two tanks 1 and 2 and the speed of tank 1 is twice that of tank 2, then the pairwise comparison matrix A will be

	1	2
1	1	2
2	1/2	1

and the new ratio scale will be the eigenvector $w = (2/3, 1/3)$ corresponding to the maximum eigenvalue 2, since $A*w = 2*w$. Note that in this case this is the same as dividing each tank's speed by the sum of the speeds.

In the preceding example, the criterion was of the type "the more the better", while a slightly different procedure must be used when the criterion is of the type "the less, the better", such as cost. The change we need is to equate the ratio of weights to the reciprocal of the ratio of costs. For example if the cost of tanks 1 and 2 are respectively s and t, then $W^2/W^1 = s/t$, and we will use the weight ratios as entries in a matrix, whose eigenvalue will give us the priorities.

Another feature of the method is that we can determine the consistency of our assessments, when the number of factors is greater than two. This is mainly applicable in cases where the criterion is not a measurable characteristic of the factors, such as comparing three cars with respect to comfort because in order to be consistent, if car A seems

one-half as comfortable as car B and one-fourth as comfortable as car C then car B must be one-half as comfortable as car C. Our judgement, however, is not always consistent and it may be important to know our consistency in order to prevent basing our decision on subjective assessments which appear to be random. That is why Saaty has defined a consistency ratio which gives us an estimate of the overall consistency of judgements. This consistency ratio is found in the following way: Given a n by n pairwise comparison matrix A and the eigenvector w which scales the factors, divide component by component the product A*w by w, and average the components of this quotient. Subtracting n to this average and dividing by 2, we obtain a consistency index for this matrix. If we choose values randomly from the set {1/9, 1/8, ...1/2, 1, 2,...9} and assign those values to the entries of a n by n matrix, we compute its consistency index. Repeating the process a reasonable number of times and averaging the consistency indexes, we will find a random consistency index. In [Ref. 23] the author gives a table of these random consistency indexes for matrices of order less than or equal 10. Dividing the consistency index of the pairwise comparison matrix A by the random consistency index of a matrix of the same order n, we find the consistency ratio of our judgements. Saaty states that a consistency ratio of 10 percent or less indicates good consistency.

With this brief explanation, we have the tools we need to use the analytic hierarchy process. In the next chapter we will use it combined with fuzzy sets to solve a decision problem and in Chapter 6 we will apply only this method to a decision making problem.

V. DECISION MAKING USING FUZZY SETS

A. DISCUSSION

In this chapter, we are going to present an application of fuzzy sets theory to a decision problem. This case will be the solution to a naval decision problem faced by the commanding officer of a naval task force. The method used, is rather simple, but is supported by the principle of incompatibility, which enunciated by Zadeh in [Ref. 21], states that ".... as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes, until a threshold is reached, beyond which precision and significance (or relevance), become almost mutually exclusive characteristics....". In other words, the traditional techniques of system analysis, become less and less suited for dealing with problems as their complexity increases. The decision making process, mainly in social systems, is so complex, that even experts in decision theory find difficults in applying it. By way of example, we can mention and quote to C. Jackson Grayson, who wrote a book in decision theory [Ref. 25], and yet, as he says in [Ref. 26], "....in the most challenging assignment of my life -putting together the Price Commission- I used absolutely none of the management science tools explicitly....".

The contention of this thesis is not that classical decision theory is better or worse than the fuzzy set theory, nor that they must be compared with the analytic hierarchy process. Rather, they are complementary and a mastering of all them will help in selecting the most appropriate way to make better decision in each specific problem at hand.

Before proceeding to present our first case, we need to review some of the concepts from fuzzy set theory that are used in decision making problems.

By a fuzzy goal, we mean an objective which can be characterized as a fuzzy set in an appropriate space. For example, if the space is the real line, we can have as a goal "maintain x substantially larger than 1000". Another frequently used example, although its space is not entirely defined, is "to improve the well-being of the people".

The most important feature to be pointed out is that there exists a symmetry between goals and constraints. That is, both concepts are defined as fuzzy sets in the space of alternatives and then can be treated identically in the formulation of a decision. This idea is similar to the way we mix objective function and constraints when using Lagrange multipliers to find the optimal solution to a constrained problem.

A fuzzy decision is defined as the intersection of fuzzy goals and constraints. That is, if we have the goals G1,G2,...Gn and the constraints C1,C2,...Cm, then the decision D is given by

$$D = G1 \cap G2 \cap \ldots \cap Gn \cap C1 \cap \ldots \cap Cm.$$

The decision, as defined, is then a fuzzy set in the space of the alternatives. Therefore we need to select from that set the alternative which has the highest membership value in D.

The following example, will help to explain the procedure, and will be used to motivate and support the changes we will use to improve that procedure. Since we have already mentioned that goals and constraints are symmetric in the space of alternatives, we can suppose, without loss of generality, the following problem.

We need to choose from the set $X = \{Joe, Dan, Bob\}$, the candidate that best satisfies the requirements that

i) The candidate should be young ,

ii) The candidate should have postgraduate studies, and

iii) The candidate should be able to communicate well, both verbally and in writing.

In this problem, X is the set of alternatives. The requirements can be identified with constraints C1, C2 and C3. If we suppose that those constraints are the fuzzy sets

$$C1 = \{(Joe, .5), (Dan, .7), (Bob, .3)\}$$

$$C2 = \{(Joe, .5), (Dan, .4), (Bob, .8)\}$$

$$C3 = \{(Joe, .2), (Dan, .5), (Bob, .6)\},$$

then the decision is the fuzzy set

$$D = \{(Joe, .2), (Dan, .4), (Bob, .3)\},$$

and the selected candidate is Dan, since he has the highest membership value.

Going over the procedure, and recalling the definition of intersection given in Chapter III, which is

$$f_{C1 \cap C2 \cap C3}(x) = \mathrm{Min}\{f_{C1}(x), f_{C2}(x), f_{C3}(x)\},$$

we see that the decision principle used with fuzzy set theory to solve decision making problems is the maximin or pessimistic principle of choice under uncertainty. This principle, considered by some to be not very attractive, is however widely used. The two following instances, will support the previous statement. The first is taken from Swalm [Ref. 27], who speaking of business decision makers, says

"....In my own studies I found that well over half of those sampled from one of the "top ten" companies, would not recommend taking a 50-50 chance of gaining $100,000 vs. losing $20,000. (Both of these sums were after tax) because if they did propose such a gamble, half the time they would have to explain a$20,000 "mistake", and if this happened too often, they might not be around to share the gains the company would, in the long run, make...."

This example shows us that the utility value of this gamble was zero for the decision makers, while the expected value for the company, given that there are many of those gambles, would be $40,000 and in spite of the prescriptive solution, that is, to take all those gambles, what these decision makers were using is the maximin principle.

Another instance where the maximin procedure is used is the two-person zero-sum games, where the player who is maximizing, assures a gain-floor, while the player who is minimizing (his losses), assures a loss-ceiling. The military doctrine of decision making in most nations, which uses the so called "estimate of the situation", is often the maximin principle of game theory.

Going back to the procedure outlined before, since it does not differentiate between goals and constraints, or from another point of view, it assumes equal importance of goals and constraints, we are going to make some changes to it. To do so, let's recall from Chapter III that if we raise a fuzzy set to some power $a>1$, the effect is to reduce the grade of membership of all the x's, but in such a way that those that have large membership values are reduced much less than those that have small values. Conversely, if $a<1$, then the membership function is increased; the smaller the value of a, the more the membership value is increased. Therefore, if we have degrees in importance for goals or constraints, then we can choose suitable exponents to decrease the membership values of those alternatives that are low valued in important goals, preventing them from

being selected as the best solution. Thus our next step consists of finding those exponents a1,a2,...an, and then we will set up our fuzzy decision as:

$$D = C1^{a1} \cap C2^{a2} \cap \; .... \cap Cn^{an} \; ,$$

where, the more important a goal, the higher its exponent. Since we cannot use negative exponents and it can occur that the goals and constraints do have equal importance in which case all exponents must be equal to one, we will put the additional conditions:

$a \geq 0$ for all i

$$(1/n) * \sum_i ai = 1.$$

In order to find these exponents, we will use the method given by Saaty. That is, we will find the eigenvector corresponding to the maximum eigenvalue of a pairwise comparison matrix. The procedure will be illustrated by the following example, which is a refinement of the previous one:

We want to chose the best candidate for a given job, and our alternatives are the elements of the (non fuzzy) set X = {Joe, Dan, Bob}, and the goals are, as before, i.e., the candidate should be young, he should have postgraduate studies, and he should be able of communicate well. Assume furthermore, that these goals are the fuzzy sets:

C1 = {(Joe, .5), (Dan, .7), (Bob, .3)},

C2 = {(Joe, .5), (Dan, .4), (Bob, .8)},

C3 = {(Joe, .2), (Dan, .5), (Bob, .6)}.

In this case, however, the goals are different in importance. Specifically, C2 is between weakly and essentially

more important than C1, therefore from Table I we find that the entry a21 of the matrix A is 4, while a12 = 1/4. For the decision maker, C3 is weakly more important than C1, therefore a13 = 1/3, a31 =3. Also C2 is weakly more important than C3 and so, a23 =3, a32 =1/3, or in matrix form

$$A = \begin{array}{|c|c|c|} \hline 1 & 1/4 & 1/3 \\ \hline 4 & 1 & 3 \\ \hline 3 & 1/3 & 1 \\ \hline \end{array}$$

and the maximum eigenvector may be found to be

w = (.1172, .6144, .2683).

Multiplying it by n = 3 (the order of the matrix), we will obtain the exponents:

a1 = .3516, a2 = 1.8432, a3 = .8409

and our fuzzy decision will be:

$D = C1^{a1} \cap C2^{a2} \cap C3^{a3}$, where

$C1^{a1}$ = {(Joe, .7837), (Dan, .8821), (Bob, .6548)},

$C2^{a2}$ = {(Joe, .2787), (Dan, .1847), (Bob, .6627)},

$C3^{a3}$ = {(Joe, .2737), (Dan, .5524), (Bob, .6628)}, and

D = {(Joe, .2737), (Dan, .1847), (Bob, .6627)}.

Now, the selected candidate is Bob, since Dan, who was the one selected in the previous example, has a low membership value (.4) in C2 which is considered the most important

goal. In this case, the decision was rather simple, given that C2 was clearly more weighted than the others. The procedure, however, will work in more complex and less transparent situations, as we present in the following section.

B. A DECISION PROBLEM

1. The Situation

As we have said, our problem is an hypothetical situation in which the commanding officer of a naval task force has to make a decision in the following situation: The mission of the task force is to conquer an island which is in enemy's hands. The task force is composed of two frigates Lupo class, and four corvettes Descubierta class, both with surface to surface and surface-to-air missiles, nine patrolboats Combatant class, and three troop transport ships LST class which transport a force of 2500 marines. It is known that the enemy has two submarines, ten patrolboats OSA class, two squadrons of aircraft with 15 aircraft per squadron, and land based surface-to-surface missiles, which are installed in the surroundings of the capital city of the island. It has been estimated that these missiles are capable of sinking the entire fleet, fortunately, their range is only 120 nautical miles. It is expected that once the marines disembark, there will be people from the island that will join the disembarked troops. Besides that, there is a reserve of troops that can be transported in a second trip of the transport ships, provided they are not sunk in the first one. The land enemy's forces, which includes nearly 9000 troops and a battalion of armored vehicles (light tanks), are mainly concentrated in the surrcundings of the capital city.

The island has an almost rectangular shape with 700 nautical miles from west to east, and 200 nautical miles from south to north, the capital city located in the north coast. Furthermore, it is supposed that the nearest point from the task force mainland is the west coast of the island. The goals Xi and constraints Yi of the commanding officer are

X1 = Conquer the capital city of the island as soon as possible.

X2 = Maintain the casualties (of ships) and fatalities at minimum.

Y1 = Lack of information about the location of the enemy's naval forces.

Y2 = Unknown amount of people willing to join the disembarked troops, but supposed to be proportional to the conquered territory.

Y3 = Since the task force does not have aircraft, and cannot receive air support from homeland aircraft, it depends upon the surface to air missiles and gunnery to defend itself against the enemy's aircraft.

The courses of action to be considered are

A : Disembark on the west coast, in which case, the naval force is likely to have no opposition, but given the distance from the capital, the enemy will present a strong resistance to the marines and the entire operation will be delayed. It will be possible, however, to transport the reserve troop to the island.

B : Disembark on the south coast, in which case the naval force will probably be attacked by the enemy's aircraft. If they are not detected early, the marines will be able

to disembark with no great opposition and their objective will be at a distance of 200 nautical miles.

C : Disembark on the north coast, just outside of the range of the enemy's surface to surface missiles (120 nautical miles), in which case both the naval and land forces will be subject to the strongest attack.

Our next step is to find out the membership functions of the fuzzy goals and constraints in the space of alternatives. Supposing we are the decision maker, we can assume that they are as follows:

	A	B	C
X1	.5	.7	.6
X2	.7	.6	.4
Y1	.6	.5	.3
Y2	.8	.7	.7
Y3	.8	.7	.6

The meaning of these values is, say for X1, that under the alternative A, goal X1 will be more or less attainable. If the alternative chosen is B, then goal X1 will be more attainable than under A. If the alternative chosen is C, then it will be attainable in between the two first.

2. <u>The Solution</u>

Our first step is to rank the goals and constraints according to their relative importance, and to do so, we have decided to use the eigenvalue method. We make the following pairwise comparisons: The first goal X1 has a weak importance over X2, therefore using Table I, we assign the

values a12 = 3, a21 = 1/3. This goal is of absolute importance over Y1, Y2 and Y3, and so, a13 = a14 = a15 = 9 and a31 = a41 = a51 = a91 = 1/9. The goal X2 is of demonstrated importance compared with the constraints, and Table I give the values a23 = a24 = a25 = 7, and a32 = a42 = a52 = 1/7. Finally Y1, Y2 and Y3 are considered of equal importance, therefore a34 = a35 = a45 =a43 = a53 =a54 =1, and the matrix A is

$$A = \begin{bmatrix} 1 & 3 & 9 & 9 & 9 \\ 1/3 & 1 & 7 & 7 & 7 \\ 1/9 & 1/7 & 1 & 1 & 1 \\ 1/9 & 1/7 & 1 & 1 & 1 \\ 1/9 & 1/7 & 1 & 1 & 1 \end{bmatrix}$$

The next step is to find the eigenvector corresponding to the maximum eigenvalue. This was found to be (.55, .3, .05, .05, .05), which multiplied by the order of the matrix, give us the exponents to use (2.75, 1.5, .25, .25, .25). The weighted goals and constrains are now

	A	B	C
X1	.148	.375	.245
X2	.586	.465	.252
Y1	.880	.841	.740
Y2	.946	.915	.915
Y3	.946	.915	.880

and the fuzzy decisicn is

D =	.148	.375	.245

Alternative B is a clear cut choice now, since it has the highest membership value in the fuzzy decision set, and the analysis suggest that the commander officer disembark on the south cost.

The procedure used can seem very simple, but looking at it from a pragmatic point of view, we can think of that simplicity as an advantage. There have been attempts to develop a so called theory of possibility, which is the fuzzy counterpart of the theory of probability, and then set up models of decision making based in the former, but they seem to be less suitable than the statistical decision theory methods. Therefore, we have prefered to present only this mcdel, which hopefully, will be useful. In the next chapter we are going to use the analytic hierarchy process exclusively to solve a decision making problem, and in the last chapter we will contrast the fuzzy set approach with the analytic hierarchy process.

VI. DECISION MAKING USING THE ANALYTIC HIERARCHY PROCESS

A. EXPLANATION

In this chapter, we are going to present an application of the analytic hierarchy process to the problem of selecting the best course of action in order to insure quality air service in the metropolitan area of Mexico City for the reminder of this century.

We must mention that though we will be very specific in the problem, it is in fact a problem shared by several large cities around the world (and others not so large as Monterey, Ca.). London has been studying for more than a decade the construction of a third airport. Meanwhile, this year they have finished the construction of a fourth terminal at Heathrow, increasing the capacity of that airport by eight million passengers per year. Bangkok and Seoul are two other cases. The former has not resolved the problem, while the latter has chosen the site for Seoul's second airport. The analytic hierarchy process seems reasonably adequate to handle problems of this type, and this application of that method attempts to show how to use it.

B. A DECISION PROBLEM

1. The Situation

In the late 60's, the Mexican government began a study to determine the most effective strategy for developing the airport facilities of the Mexico City's metropolitan area. The objective was to insure quality air service for the reminder of the century. The study was performed

jointly by the Center for Computation and Statistics and the Department of Airports, both part of the Ministry of Public Works (Secretaria de Obras Publicas). Professors Richard de Neufville and Ralph L. Keeney from the Massachusetts Institute of Technology and Howard Raiffa from Harvard University were consultants assisting in the study. The results of that study are reported by the first two [Ref. 28], and also by Keeney alone [Ref. 29]. In that study, two basic alternatives were considered:

1) Expand the existing airport located in Texcoco, or
2) Build an additional new airport in a valley called Zumpango located 40 kilometers north of Mexico City, and do not expand the existing airport.

The users of the airport's services were classified as international (I), domestic (D), general (G), and military (M). This classification expanded the set of alternatives, and in fact the group of consultants received a fixed set of alternatives to work with. These alternatives were all possible combinations in assigning the users (I,D,G,M) to the two proposed locations. For example one alternative would be to send users G and M to Zumpango with I and D remaining at Texcoco. This alternative was represented as (T-ID, Z-GM).

Since there were four types of users, each of which could be assigned to one of the two locations, the number of alternatives was 16. Furthermore, the study considered three points in time. Specifically, the assignment of users was done for 1975, 1985 and 1995. Thus the set of alternatives would include all possible but logical combinations of the assignment of users to locations in those three years. This meant that out of the 4096 ((2**4)**3) possible alternatives, they excluded those that were impractical, such as to move all users to Zumpango in 1985 and go back to Texcoco in 1995. In [Ref. 29], Keeney says "....In the final analysis,

the total number of alternatives evaluated was approximately 100....".

The consultants' recommendation for immediate action in 1971, when the study was ended [Ref. 28] pp. 517, was: "....At Zumpango, do no more than buy land for an airport. At Texcoco, extend the two main runways and the aircraft apron, construct freight and parking facilities and a new control tower. Do not build any new passenger terminals...". In the other report, pp. 114, the highest ranked alternative for the horizon of the study was to move the domestic users to Zumpango by 1975, to move the international users to Zumpango by 1985, and to let general and military users remain in Texcoco indefinitely. The second ranked alternative was to move all users to Zumpango by 1975.

The actions taken by the Mexican government in the middle 70's were to build a new control tower, extend the runways, and enlarge the passenger facilities of the existing airport located 9 kilometers east of the center of the city. The long term solutions recommended were not implemented, since the construction of the new airport has not been decided. Besides that, the conditions existing at the time of the study of reference have changed radically. To begin with, the population of Mexico City, which grew from 5 million in 1960 to 8 million in 1971, is now 16 million [Ref. 35].

Consequences of this population growth related to the problem we are looking at are several, such as insufficient capacity of the passenger facilities for peak demands, a greater quantity of people subject to high levels of noise, and high levels of air pollution. Another factor to consider is the current economic position of Mexico. Any investment that would require the purchase of foreign equipment would be subject to great scrutiny, unless it was considered absolutely necessary or strategically important.

With this background we wish to demonstrate the use of the analytic hierarchy process as a tool to answer the questions:

Should the Mexican government begin the construction of the Zumpango airport?

If so, how should the users be assigned to both airports?

A word must be said before proceeding. The opinions and statements we are going to use do not reflect the official position of the Mexican government and can be considered, at most, as those of concerned citizens. The same may be said of our subjective assessments of the importance of one factor over other, in the pairwise comparisons required by the method. This will not be a benefit cost analysis, although that is possible using the analytic hierarchy process, because such analysis would require us to take into account all the projects proposed to the Mexican government, which will compete for the scarce resource of the federal budget. Therefore we will concentrate on the problem as stated previously, and we will follow the steps outlined by Saaty in applying the analytic hierarchy process.

2. <u>The Solution</u>

The first step is to define the problem. In our case, and given that the Mexican government had stated it, we will define it as:

Given the current situation of the country in general, and the existing Mexico City's airport in particular, what is the best way to insure quality air service for the reminder of this century in the metropolitan area of Mexico City?.

The elements or factors to be considered in the problem need now to be structured in a hierarchy, but of course, we need first to identify those factors and to

determine the level in which each should be placed. As we have said in the first part of this chapter, the top level of the hierarchy or level one consists of our overall objective, and we will refer to it as the focus. The following level should consist of those factors which have a direct impact on the focus and can be compared pairwise using the focus as the criterion.

It seems reasonable that the second level of our hierarchy should include the same measures of effectiveness used in the study of reference, together with the air pollution factor which was neglected there. The second level elements are:

1. The cost of the alternative in millions of Mexican pesos.
2. The capacity in millions of passengers per year.
3. The distance from the center of the city in kilometers.
4. The expected number of fatalities per year (including non passengers) due to aircraft accidents while landing or taking off.
5. The average number of people subject to a high noise level (90 composite noise rating).
6. The increment in the Mexico City's air pollution due to the airport's operations.

Factors Number 1, 2, 3, and 4 are self explanatory, while Numbers 5 and 6 need a comment. The sources of noise in the airport' operations include taking off, landing and taxi maneuvers of aircraft, various ground support equipment, and engine testing in a test cell.

The units used to measure the noise are the Composite Noise Ratio (CNR), the Noise and Number Index (NNI) used in England, and a more sophisticated measure called the Noise Exposure Forecast (NEF). Large and Lams in [Ref. 32] pp. 141, state that the NNI unit is more useful when designing a new airport, while CNR and NEF, the latter being a modification of the former, are more appropriate for solving the noise problem in existing airports.

We have decided to use the CNR unit, since the Zumpango valley, where the new airport would be constructed, is far from inhabited areas, and it is the existing airport in Texcoco which has noise as an increasing problem.

The air pollution consists mainly of five types of pollutants which are monoxide of carbon, hydrocarbons, oxides of nitrogen, particulates, and lead. Only the first four have been found to be produced by an airport's operations and of the lbs/day of CO produced by all sources in Washington D. C., 2.3 percent was produced by aircraft operations [Ref. 38]. The percentage for hydrocarbons is 1.4 percent , the percentage for NO is 0.43 percent , and for particulates is 0.69 percent. These same values were found in Monterey CA. [Ref. 39]. Although the impact of the airport's operation in the air pollution is relatively small, we will retain that factor in the structure of the hierarchy, since it is for a practice.

The next level of the hierarchy consists of the alternatives we have selected as feasible solutions to our problem, and they are those that were ranked highest in the study of reference:

1. Construct the Zumpango airport and move all users there as soon as it is ready to operate. This alternative will be identified as (Z-IDGM).
2. Construct the Zumpango airport and transfer the domestic and international users as soon as it is operational. This alternative will be identified as ((Z-ID, T-GM).
3. Construct the Zumpango airport and transfer the domestic, general and military users there. This alternative will be (Z-DGM, T-I).
4. Construct the Zumpango airport without passenger facilities and transfer the general and military users there. Enlarge the facilities at Texcoco. This alternative will be refered to as (Z-GM, T-ID).

5. Do nothing at Zumpango and expand the existing airport as much as possible with the available space. Enhance the efficiency in airport management, fix stricter limits in noise emission by aircraft and take the necessary measures to optimize the utilization of the airport facilities. This alternative will be (T-IDGM).

The hierarchy, as we have structured it, is shown in Figure 6.1

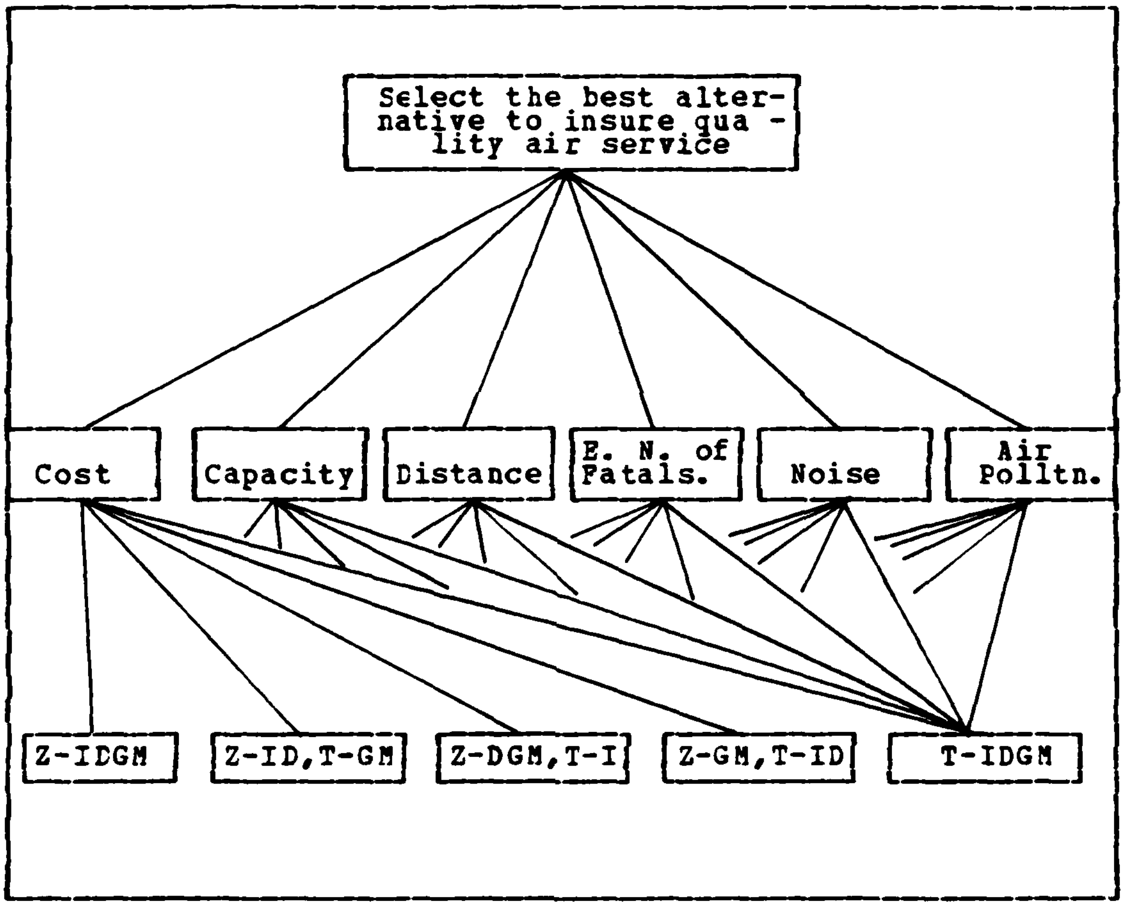

Figure 6.1 Structure of the Hierarchy for Case 2.

The next step is to make the pairwise comparisons between the factors of Level 2, using the focus as criterion. To do this, we need first to compare the importance we give to cost as compared with all other factors of Level 2. Due to the financial situation of the country, cost is considered strongly important and so using the scale given in Table I we will assign to the entries a12, a13, a14, a15, and a16 values of 5.0 for 'essential or strong importance' of this factor as compared with all others, that is a12 = a13 = a14 =a15 = a16 = 5.0 . Capacity is considered slightly mcre important than distance, equal important than the expected number of fatalities, slightly less important than the noise and the air pollution, so we will set a23 = 3, a24 = 1, a25 = 1/2, and a16 = 1/3. The distance seems slightly less important than the expected number of fatalities, the noise, and the air pollution, so a34 = 1/2, a35 =1/3 and a36 = 1/2. The expected number of fatalities might be considered a little more important than the noise and the air pollution, so a45 = a46 = 2. The noise and the air pollution will be considered equally important so a56 = 1. Symmetric entries to those given are reciprocals of the values assigned, and thus the pairwise comparison matrix is

		Cost 1	Capa-city 2	Dist. 3	E.Num. of F. 4	N. 5	Air Polltn. 6
Cost	1	1	5	5	5	5	5
Capacity	2	1/5	1	3	1	1/2	1/3
Distance	3	1/5	1/3	1	1/2	1/3	1/2
E.N. of Fat	4	1/5	1	2	1	2	2
Noise	5	1/5	2	3	1/2	1	1
Air Polltn'	6	1/5	3	2	1/2	1	1

The eigenvector corresponding to the maximum eigenvalue of this matrix, gives us a ratio scale of the importance of these factors for the decision maker, that is, the priorities of the factors obtained are

1 - .477

2 - .090

3 - .054

4 - .140

5 - .115

6 - .121.

The consistency index of this matrix is 0.0926 and the random consistency index for a 6 by 6 matrix is 1.24, so the consistency ratio is 7.46 percent, which is considered adequate.

Following the procedure outlined before, we need now to find the priorities or weights of the five alternatives, taking as criteria each of the six factors of Level 2. Of the six factors only capacity is of "the more the better" type. All others are of "the less the better" type.

Each alternative can be compared directly with the others with respect to the factors of Level 2, since we can compare any two alternatives in their costs using pesos, in their capacities in millions of passengers per year, etc. We have seen also at the begining of this chapter that in order to determine the priorities of alternatives with respect to measurable factors, we can use either the algebraic or the eigenvalue method.

The rating of the alternatives with respect to cost may be done using the following rationale: Let the cost of our first alternative identified as (Z-IDGM) be C1.[1] Since our second alternative differs of the first slightly we have

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[1]The most recent estimate for the cost of a new airport is that reported in [Ref. 34] for the planned new airport in Bangkok, which will cost two billion dollars.

estimated that the cost of the second is approximately 85 percent of the cost of the first. That is, C2 = 0.85*C1, and since the ratio of the weights or priorities assigned to each alternative is the reciprocal of the ratio of costs, we have W2/W1 = C1/.85*C1 or W1/W2 = .85, and this is the entry a12 of the pairwise comparison matrix.

Estimating the cost of alternatives 3, 4, and 5 as a percent of the cost of alternative 1, we can find the ratios of the weights of the first alternative with all others, and these ratios will be the entries of the first row of the pairwise comparison matrix. Using the relation (W1/Wi)*(Wi/Wj) = (W1/Wj), which is (a1i)*(aij) = (a1j), we can find all other entries of the matrix, since (aij) = (a1j)/(a1i). We recall also that aii = 1, and aij = 1/aji. So the matrix which is the result of comparing our alternatives, with respect to cost is:

COST		ALTERNATIVE 1	2	3	4	5
ALT.	1	1	.85	.5	.2	.1
"	2	1.176	1	.58	.23	.12
"	3	2	1.7	1	.4	.2
"	4	5	4.255	2.5	1	.5
"	5	10	8.470	5	1.996	1

and the ratio scale of the alternatives with respect to cost is given by the eigenvector

(.0521, .06138, .1042, .261, .5210).

Using the same rationale, but now equating the ratio of capacities to the ratio of weights, gives us

CAPACITY		ALTERNATIVE 1	2	3	4	5
ALT.	1	1	1	1	2	2
"	2	1	1	1	2	2
"	3	1	1	1	2	2
"	4	.5	.5	.5	1	1
"	5	.5	.5	.5	1	1

and the vector of priorities is
(.25, .25, .25, .125, .125).

In order to find the ranking of the alternatives according to their distance to the center of the city, the rationale used is as that we used for cost except that we introduced a small variation in the ratio of distances, to take into account the density of traffic on the roads used to reach the airports. Alternative 3 (Z-DGM, T-I), has been assigned the value .5, given that the domestic users will have to travel to Zumpango, while the international will go to Texcoco. The matrix is

DISTANCE		ALTERNATIVE 1	2	3	4	5
ALT.	1	1	1	.5	.25	.25
"	2	1	1	.5	.25	.25
"	3	2	2	1	.5	.5
"	4	4	4	2	1	1
"	5	4	4	2	1	1

and the vector of pricrities is
(.083, .083, .167, .334, .334).

The rationale for the other three factors, i.e., expected number of fatalities, noise and air pollution, is the same as the preceding, with the exception that in those cases, the largest value corresponds to alternative number 5 (T-IDGM). Therefore, in those three cases, we are going to fill the fifth row of the matrix using the direct comparisons between alternatives. Another feature is that we can use only one matrix for the three factors, since they are proportional to each other, i.e., the higher the noise the higher the air pollution, etc,. The matrix is

FACTORS 4, 5, 6		ALTERNATIVE 1	2	3	4	5
ALT.	1	1	2.99	2.99	6.99	10
"	2	.3333	1	1	2.333	3.333
"	3	.3333	1	1	2.333	3.333
"	4	.14	.428	.428	1	1.42
"	5	.1	.3	.3	.7	1

and the eigenvector which gives us the ratio scale of the alternatives with respect to these factors is
(.523, .174, .174, .074, .052).

We must point out that the 6 pairwise comparisons matrices of the alternatives with respect to each of the factors of level 2, have a consistency ratio negligible.

Finally we are ready to obtain the composite priorities of the alternatives. As it was said before, this is accomplished by weighting each vector of priorities by the priority of the criterion and summing these weighted vectors. Table II shows the results of that operation.

TABLE II

Composite Priorities of Alternatives

	A L T E R N A T I V E S				
	Z-IDGM	Z-ID T-GM	Z-DGM T-I	Z-GM T-ID	T-IDGM
	1	2	3	4	5
Cost (.477)	.05*.47	.06*.47	.10*.47	.26*.47	.52*.47
capa city (.09)	.25*.09	.25*.09	.25*.09	.13*.09	.13*.09
dis tance (.054)	.08*.05	.08*.05	.17*.05	.33*.05	.33*.05
fata lities (.140)	.52*.14	.17*.14	.17*.14	.07*.14	.05*.14
Noise (.115)	.52*.12	.17*.12	.17*.12	.17*.12	.05*.12
air polltn. (.121)	.52*.12	.17*.12	.17*.12	.17*.12	.05*.12
Comp. Prits.	.248	.122	.146	.182	.297

These composite priorities are a ratio scale of the alternatives and the alternative with the highest value is alternative number 5, that is, the one identified as

(T-IDMG). The best solution is then, not to build the new airport but to look for a means to enhance the efficiency of the existing airport. It is important to note that the second best alternative is to build the new airport and transfer all users there. In summary, the conclusion from our estimated values is that the best thing the Mexican government can do right now, is to use all means at hand to give a good air service in the existing airport.

In order to see the effect that a change in our estimated values will produce in the conclusion reached, we changed the importance of the factor cost, as would happen if the recovery of the Mexican economy released the high priority given to that factor. To do so we assign a value of 3 which corresponds to a weak importance of factor cost over all others to entries a12, a13, a14, and a15. Since the effect of the airport's operations in the air pollution is small, we deleted that factor and the pairwise comparison matrix is

		Cost 1	Capacity 2	Dist. 3	E.Num of F. 4	N. 5
Cost	1	1	3	3	3	3
Capacity	2	1/3	1	3	1	1/2
Distance	3	1/3	1/3	1	1/2	1/3
E.N. of Fat	4	1/3	1	2	1	2
Noise	5	1/3	2	3	1/2	1

and the ratio scale of these factors is now

1 - .4036

2 - .1385

3 - .0764

4 - .1847

5 - .1966

with a consistency index of 0.1082, and since the random consistency index of a 5 by 5 matrix is 1.12, we have a consistency ratio of 9.6 percent, which is acceptable.

Using these values in a computation as the one shown in Table II, we found the following values for the composite priorities of the alternatives

Alternative 1 = (Z-IDGM) .2614

Alternative 2 = (Z-ID,T-GM) .132

Alternative 3 = (Z-DGM,T-I) .1557

Alternative 4 = (Z-GM,T-ID) .1763

Alternative 5 = (T-IDGM) .272

Alternatives 1 and 5 are now very close to each other and to see more clearly this trend we reduced still more the importance given to the cost assigning a value of 2 to the entries of the first row of the pairwise comparison matrix, except a11, and the composite priorities of the alternatives were

Alternative 1 = (Z-IDGM) .2935

Alternative 2 = (Z-ID,T-GM) .1428

Alternative 3 = (Z-DGM,T-I) .1635

Alternative 4 = (Z-GM,T-ID) .1628

Alternative 5 = (T-IDGM) .2337

Alternative 1 has now the highest value, while alternative 5 which was the highest is now the second.

With this example, we have attempted to demonstrate the usage of the analytic hierarchy process in a decision making problem. In the following chapter we will compare this method with the fuzzy set approach, in order to find cut their similarities and differences.

VII. CONCLUSIONS

The two techniques we have used share the common characteristic that they were developed to handle complex problems in a simple but structured way, which was a goal of their authors. Zadeh in [Ref. 21], says that in order "....to deal with humanistic systems realistically, we need approaches which do not make a fetish of precision, rigor and mathematical formalism, and which employ instead a methodological framework which is tolerant of imprecision and partial truths....". Fuzzy set theory, he says, "....is a step - but not necessary a definite step - in this direction....". In this regard, Saaty, author of the analytic hierarchy process says "....what we need is not a more complicated way of thinking,... Rather, we need to view our problem in an organized but complex framework that allows for interactions and interdependence among factors and still enables us to think about them in <u>a simple</u> way ....".

In the case we solved using fuzzy sets, we found that the alternative chosen was the one with higher membership value in the main goal. In a more complex problem where we could have multiple and possibly conflicting objectives, the technique would be able to weight goals and constraints proportionally to their importance to the decision maker. The weak point of the technique is how to determine the membership value of the fuzzy alternatives in each fuzzy goal or constraint. Unfortunately this point will depend heavily upon the experience and judgment of the decision maker. In this regard McNamara said that"....Granted there are specific techniques, facts and calculations involved; in the final analysis, judgment is what is at issue...". In other words, decision aids are just that and no more. This

does not contradict a statement we made in the sense that we need analytic techniques to help the decision maker, since those techniques will clarify the problem to him, enabling a more comprehensive sclution.

Another important feature to be pointed out is that both techniques seem more suitable for nonrepetitive decision problems, and we must be aware of this when dealing with such problems. In those cases using the expected utility method would be more appropriate, as we saw in Chapter Five in the example given by Swalm [Ref. 27].

In any case, both techniques are on their way to becoming popular, since for example there are microcomputer software packages, available for aid in decision making based in the analytic hierarchy process, and there are interactive packages running in mainframes which uses the fuzzy set approach.

At this point we want to stress the importance of two factors in any decision making problem. The first is is the quality, quantity and reliability of the information about the problem. Some authors believe that the quality of the solution to a decision problem is bounded by the information we have, and in fact we can estimate the cost of the information using classical decision theory. The second factor is the horizon to be considered in the sense that some solution for the short term, could not be the long term solution and it is reasonable to give more importance to the latter than that given to the former.

We will conclude this paper outlining a procedure to use in order to solve a problem in the most general case. This procedure may be used whatever a decision is precipitated by a problem of one sort or another. The first step in making any important or worthwhile decision is to define the problem. An accurate definition of the problem is already a major step toward its eventual solution. To do so we need to

gather background information in the general area under consideration. Once the problem has been defined, the definition should be carefully examined to determine the degree to which it is a true statement of the problem.

The second step is to identify the alternatives. The importance of this step needs to be stressed. It can happen that looking for the solution to the decision making problem, may consist of a search for a satisfactory alternative. The alternatives may be obvious or may not, but an effort must be made to avoid overlooking a reasonable alternative. It is a good practice to write down all possible alternatives to the problem solution, no matter how foolish they may seem at first.

The following step is to quantify the alternatives found in the previous step. Is in this step where we can use the analytical techniques mentioned or used in this paper. Since all decision aids rely on the availability of precise information, using this techniques may prod the decision maker to understand more fully the scope of the problem, the differences among alternatives, and the solution to the problem. Once the decision has been made appropriate actions must be taken to ensure that the decision must be carried out as planned

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